

# Charged Particle Motion in EM-Fields

## Tutorials for NX-Magnetics

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THIS SIMULATION SOFTWARE USES FINITE ELEMENT METHODS. USERS SHOULD BE AWARE THAT RESULTS CAN HAVE UNPREDICTABLE ERRORS IF INPUT DATA IS NOT COMPLETELY CORRECT. THEREFORE ANY DESIGN DECISIONS SHOULD NOT BE BASED SOLELY ON THE SIMULATION. USE ADDITIONAL MEASUREMENTS TO ENSURE THE CORRECTNESS.

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# 1 Theory

In the following section we will provide a short introduction into the physics of charged particles. Here, we will first provide an abbreviated description of the continuous system and then outline the basic numerical simulation techniques.

## 1.1 Continuous Description

Let us first provide an overview on the principle interaction behaviour of charged particles. To this end let us describe the interplay between particles and electromagnetic fields; as well as many particle phenomena.

### 1.1.1 Charged Particles

We start with the concept of a point-particle, sometimes called Dirac particle. A point-particle is a rather theoretical concept, describing a particle that occupies exactly on point in phase-space (i.e. it is located at an exact location  $\mathbf{x}$  with exact velocity  $\mathbf{v}$ ) having a Mass  $m$ . Therefore, the point-particle concept is a principle concept o classical mechanics. Its motion is described by Newtons second law

$$\mathbf{F} = m \mathbf{a}, \quad (1)$$

where  $\mathbf{F}$  is a principal newtonian force. If the above particle also comprises an electric charge  $q$ , thus making it an charged point-particle, we can specify the acting force to be the Lorentz force

$$\mathbf{F}_L = q (\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (2)$$

where  $\mathbf{E}$  and  $\mathbf{B}$  are the electric field and the magnetic flux density, respectively. Indeed, basic physic tells us that a charged particle is accelerated by an electric field and deflected by an magnetic field (this is exactly the meaning of the Lorentz Force). With this knowledge at hand we can now derive the velocities and the positions by simple integration

$$\mathbf{a}(t) = \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (3)$$

$$\mathbf{v}(t) = \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) t + \mathbf{v}_0, \quad (4)$$

$$\mathbf{x}(t) = \frac{1}{2} \frac{q}{m} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) t^2 + \mathbf{v}_0 t + \mathbf{x}_0. \quad (5)$$

Observe the immanent charge-mass relation factor  $\frac{q}{m}$  that determines the coupling strength.

### 1.1.2 Maxwells Equations with Charged Particles

With the principal particle description at hand, we may now continue with the field description. We know that electromagnetic fields are described by Maxwell's equations

$$\nabla \cdot \mathbf{D} = \rho, \quad (6)$$

$$\nabla \times \mathbf{H} = \frac{d\mathbf{D}}{dt} + \mathbf{J}, \quad (7)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (8)$$

$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}. \quad (9)$$

We can split this set of coupled equations into two parts: source equations (i.e. Gauss' Law and Ampere's law) and source-free equations (i.e. Faradays Law and magnetic Gauss law). From the previous discussion we know that a particle  $i$  carries a charge  $q_i$ . Moreover, moving charges also create a current (microscopically speaking this is the actual origin of currents: the macroscopic current in a wire is created by multiple moving electrons)

$$\mathbf{J}_i = q_i \mathbf{v}_i. \quad (10)$$

With this in mind we can directly write down macroscopic charges and currents

$$\rho = \sum_i q_i \quad \text{and} \quad \mathbf{J}_{\text{part}} = \sum_i q_i \mathbf{v}_i. \quad (11)$$

Here,  $\rho$  describes an ensemble of point-particle charges and directly provides a source for Gauss' law. If one combines the particle current with the possible conductor current, i.e.  $\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}_{\text{part}}$ , the resulting total current  $\mathbf{J}$  may be used as a (current) source for Ampere's law.

### 1.1.3 Vlasov Equation and Plasma Physics

## 1.2 Discrete Description

### 1.2.1 Particle In Cell Methods

The PIC method for electric and magnetic field calculations consists of the following procedure. Discrete charge density and current density arrays are defined with values in each cell. The charge density in a cell at time  $t$  equals the product of the number of computational particles in the cell times the charge per computational particle divided by the cell volume. The current density is the average vector velocity of computational particles multiplied by their charge divided by the cell area.

The space-charge and current density functions are combined with a Finite Element solution of the Maxwell equations to generate electric and magnetic field values. The PIC procedure involves one pass through the array of computational particles assigning them to cells, followed by a field computation. The number of mathematical operations is linearly proportional to the number of particles and the number of cells. In a simulation involving thousands of particles, the PIC process takes much less time than a direct evaluation of inter-particle forces.

An electrostatic computer simulation using the PIC method for field calculations consists of the following operations at each time step:

1. At time  $t$ , the electric field at the location of each computational particle is interpolated from the field result. The field is used to advance the vectors  $\mathbf{x}(\mathbf{t})$  and  $\mathbf{v}(\mathbf{t})$  for the particles to time  $t + \Delta t$  using an accurate difference scheme.
2. The charge function is evaluated at the mesh points by assigning the charge of the computational particles according to their position.
3. The electrostatic equation is solved to find electric fields at  $t + \Delta t$ , and the process is repeated.

The procedure continues until the beam advances to the desired final state.

### **1.2.2 Particle Stepper**

### **1.2.3 Particle Smearing**

### **1.2.4 The Testparticle Method**

## **1.3 Fixed Source Emission Model (Type 0)**

The fixed source emission model allows defining fixed velocities and particle numbers to emission geometry.

## **1.4 Given Velocity Field Emission Model (Type 3)**

The 'Given Velocity Field' emission model allows defining particle number and initial velocities through a tabular field. This can be for instance the velocities resulting from CFD simulations.

## **1.5 Thermal Emission Models**

Thermionic emission is the thermally induced flow of charged particles from a surface. This occurs because the thermal energy given to the particles overcomes the escape energy of the material. The charged particles can be for instance electrons or ions. The classical example of thermionic emission is that of electrons from a hot cathode into a vacuum (also known as thermal electron emission or the Edison effect) in a vacuum tube. The hot cathode can be a metal filament, a coated metal filament, or a separate structure of metal. The below table lists typical escape energies and work temperatures of some materials. Following are thermionic source types explained that are available in MAGNETICS.

	Material	Escape Energy Wa [eV]	typical work temperature [K]
<b>Metal cathodes</b>	Cäsium	1.94	
	Molybdän	4.29	
	Nickel	4.91	
	Platin	5.30	
	Tantal	4.13	2400
	Thorium	3.35	
	Wolfram	4.50	2600
<b>Metal film cathodes</b>	Barium film on Wolfram	1.5-2.1	1200
	Cäsium film on Wolfram	1.4	
	Thorium film on Wolfram	2.8	2000
	Barium film on Wolframoxide	1.3	
<b>Oxide cathodes</b>	Barium oxide	1.0-1.5	
	Barium oxide with Strontium oxide	0.9-1.3	1100

Table: Experimentally determined work functions for the thermal emission of electrons from hot cathodes (Krieger data)

### 1.5.1 Ideal Gas with mean velocities (Type 1 and 5)

The number of particles is given by the user. For the computation of initial velocities  $v$  there are two options possible:

1. Type 5: Ideal Gas with mean velocities, local temperatures: Velocities are computed by the mean of Maxwell Speed Distribution for ideal gas which is given as

$$v = \sqrt{\frac{3kT}{m}}$$

with  $k$  the Boltzmann's constant,  $m$  the mass of an particle and  $T$  the locally varying temperatures which can be applied as field or precomputed by thermal simulation.

2. Type 1: Random Velocity at a fixed (global) Temperature: Velocities are computed through the same mean of Maxwell Speed Distribution for ideal gas as for type 5, but directions are randomly defined. Temperatures are not local but fixed by the user (global).

### 1.5.2 Richardson-Dushman (Type 6 and 7)

Richardson's equation describes the current density  $J$  of electrons emitted from a metal at high temperatures. it is

$$J = AT^2 e^{-W/kT}$$

where

$$A = \frac{4\pi emk^2}{h^3} = 1.2 \times 10^6 \text{ A/m}^2 \text{ K}^2$$

Here,  $T$  is the temperature,  $W$  the work functions (escape energy) for electrons,  $k$  the Boltzmann constant and  $A$  the Richardson constant. This source type assumes that the electron current emitted by an electrode is independent of the applied voltage, and that it depends only on the

temperature, work function and emission constant of the material. This is also called thermal saturation limit. The number of particles and the initial velocities are both derived automatically if this type is chosen. Temperatures can be locally varying, applied as initial constraint, field or precomputed by another simulation.

Initial velocities  $v$  are derived from the escape energy by

$$v = \sqrt{\frac{2W}{m}}$$

with  $W$  the escape energy in electron volt and  $m$  the particle mass. There are two options available that control the velocity directions:

1. Type 6: Directions Orthogonal
2. Type 7: Random directions

The user should chose the input parameters (temperature and escape energy) carefully because in many cases there will be either none or a huge number of particles created.

### 1.5.3 Child's Law (Type 8 and 9)

Child's law (or Child, Langmuir) gives the maximum current density that can be carried in a beam of charged particles across a one dimensional accelerating gap. In order to apply this equation within the program, an accelerating gap width  $d$  and a voltage difference  $V_a$  must be applied. The equation only applies to infinite planar emitters. For electrons, the current density  $J[A/m^2]$  is written:

$$J = \frac{I_a}{S} = \frac{4\epsilon_0}{9} \sqrt{2e/m} \frac{V_a^{3/2}}{d^2}$$

where  $I_a$  is the anode current and  $S$  the anode surface inner area;  $e$  is the magnitude of the charge of the electron and  $m$  is its mass.

Particle velocities  $v$  are calculated after time step 1 with the assumption of a constant electric field  $E$  and energy conservation through Newton's law  $F = ma$  and Lorentz force  $F = qE$  what results as

$$v = \frac{q}{m} E \Delta t$$

Using those velocities  $v$  at time step 1 and  $J$  from Child's law the number of particles  $n$  can be computed with  $J = nqv$ .

Finally the initial velocities and particle positions must be found. There are two options available for this:

1. Type 8: Zero Initial Velocity: Particles are initialized with zero velocity at time step 0 at the source elements.
2. Type 9: Non Zero Initial Velocity: With the knowledge of velocity and position at time step 1 particles are propagated to time step 0. The resulting initial velocities are non zero and initial positions are near to their emission surfaces.

## 1.6 References

- Roger W. Hockney, James W. Eastwood: Computer Simulation Using Particles. CRC Press, 1988
- Jones and Langmuir GE Review, The Characteristics of Tungsten Filaments as Functions of Temperature 30 (1927) Part I pp. 310–19, Part II pp. 354–61, Part III pp. 408–12
- Melissinos Experiments in Modern Physics, 1966, pp. 65–80
- Preston and Dietz The Art of Experimental Physics, 1991, pp. 141–47, 152–61
- Blakemore Solid State Physics, 1974, pp. 188–95
- Koller The Physics of Electron Tubes, 1937, Ch. I, VI, VIII
- Script "Thermionic Emission" Chapter 5.  
Found at <http://www.physics.csbsju.edu/370/thermionic.pdf>
- Hanno Krieger: "Strahlungsquellen für Technik und Medizin"



## 2 Tutorial 1

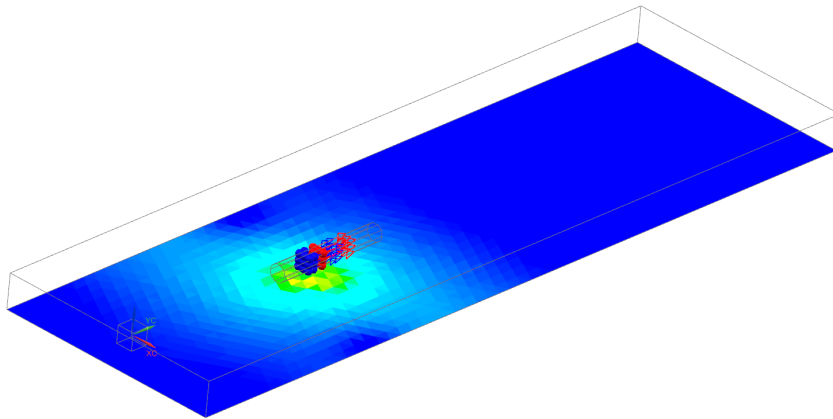
### 2.1 Particle Accelerator Pipe

The following three examples are intended to show the principal functionalities of the **Particle Solver** in the context of a linear particle accelerator. In particular the flexibility of the Solver will be demonstrated, in the context of a Magnetodynamic, a Magnetostatic and an Electrostatic simulation. The three examples are based on each other, and should thus be executed in consecutively.

#### 2.1.1 Bunched Particles – Magnetodynamic

Abstract: A bunch of particles starts with a velocity of  $15 \frac{\text{mm}}{\text{s}}$  from the displayed particle source and is headed towards an accelerator pipe with a constant electric field of  $1 \frac{\text{V}}{\text{mm}}$ , in order to be linearly accelerated. Both the displacements and the velocities will be evaluated and compared to calculated values.

Estimated time: 0.5 h Follow the steps:

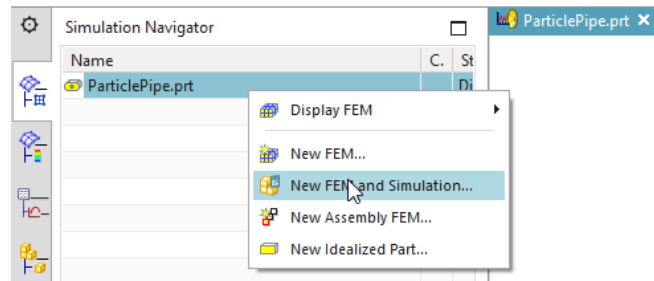


1. Download the model files for this tutorial from the following link:  
<https://www.magnetics.de/downloads/Tutorials/11.CouplParticle/11.1ParticlePipe.zip>
2. Open the file 'ParticlePipe.prt'.
3. Observe the dimensions of the particle source (red), the accelerator pipe (green) and the air (blue).
  - The particle source is a cube comprising 15 mm edges;
  - The accelerator is a cylinder of length 100 mm (and radius 15 mm);
  - The accelerator rear face is distanced 150 mm from the origin; and 142.5 mm from the front face of the source;
  - The accelerator front face is distanced 375 mm from the front boundary of the surrounding air box.

⇒ In summary the three vital distances are 100 mm, 150 mm and 375 mm.

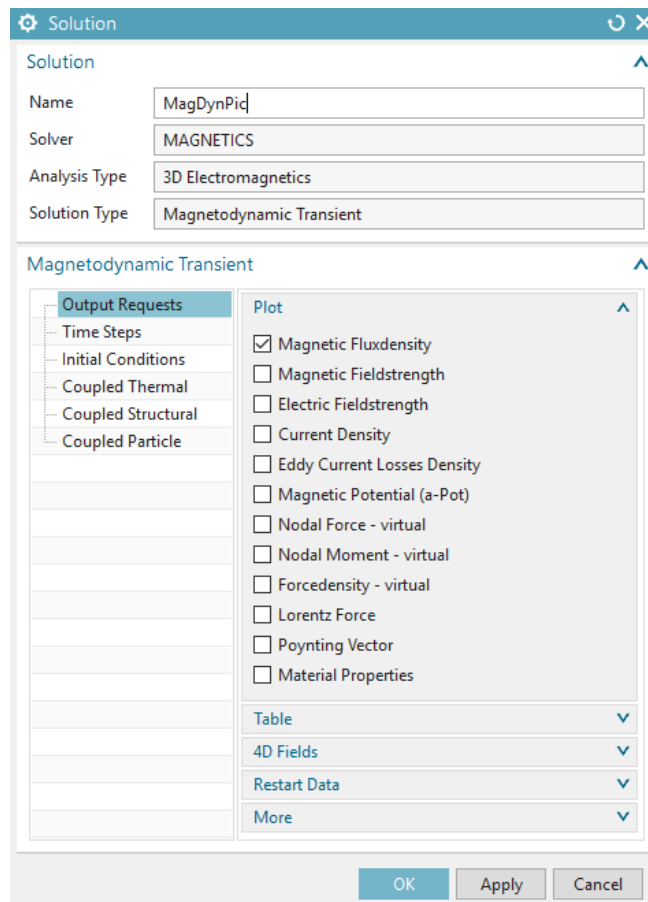
4. Switch to SIMCENTER PRE/POST

5. Create a new FEM and Simulation. Choose Solver MAGNETICS and Analysis Type '3D Electromagnetics'. Switch off the 'Create idealized Part'.

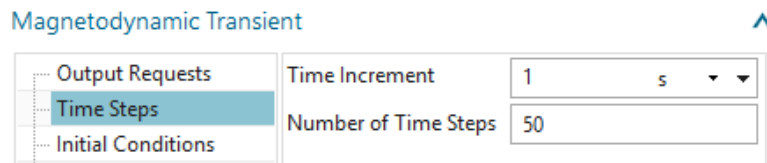


6. Choose Solution Type 'Magnetodynamic Transient'.

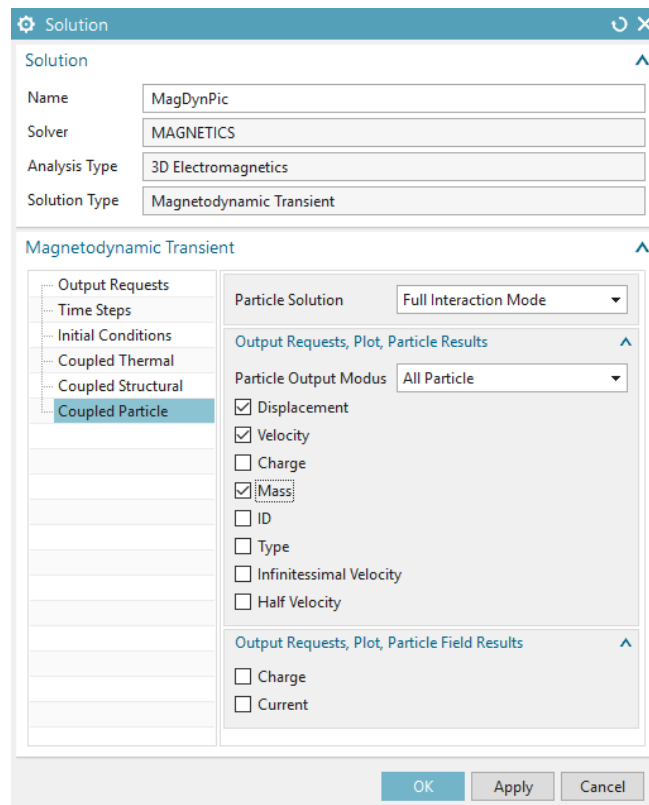
- In register 'Output Requests' under 'Plot' activate 'Magnetic Fluxdensity' to enable the calculation of the B field. Moreover, the Magnetic Potential (a-Pot) and 'Electric Fieldstrength' might be interesting (but not vital) for this tutorial.



- Under 'Time Steps' select a 'Time Increment' of 1 s; and under 'Number of Time Steps' a total of 50 time steps.



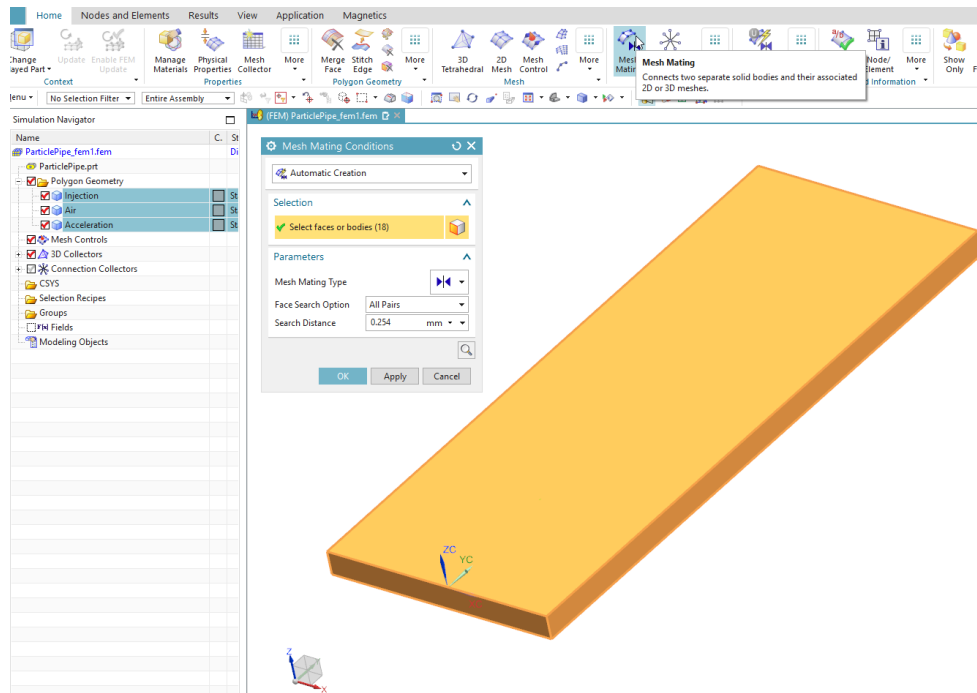
- In register 'Coupled Particle' set the 'Particle Solution' to 'Full Interaction Mode'. With this selection the Particle Solver is bi-directionally coupled with the field solver, meaning that the EM fields accelerate/deflect the particles; and in turn the particles modify the sources and currents for the field solver (for more information THEORY). Also set the 'Particle Output Mode' to 'All Particle' and select 'Displacements', 'Velocity' and 'Mass' as Output.



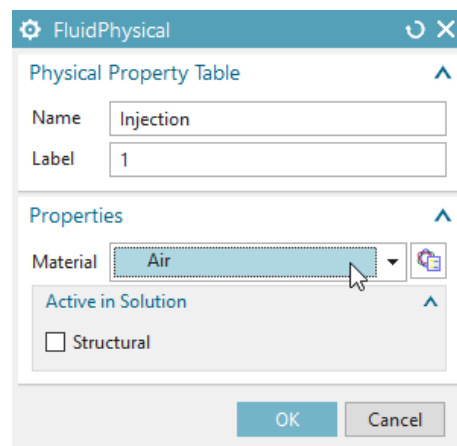
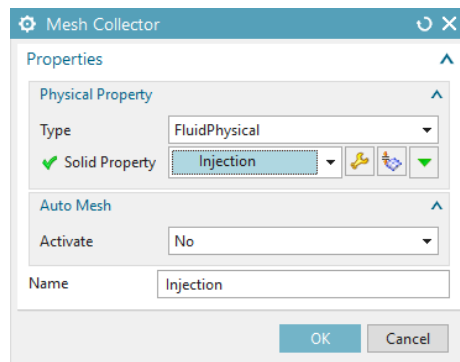
- In register 'Initial Conditions' accept the default magnetic condition 'Zero Vector Potential'.
- Press 'Ok'.

7. Next, switch to the FEM environment and make it a 'Displayed Part' (right click on the ".fem" file)

- First, create 'Mesh Matings' for all three bodies; resulting in 9 MMC.

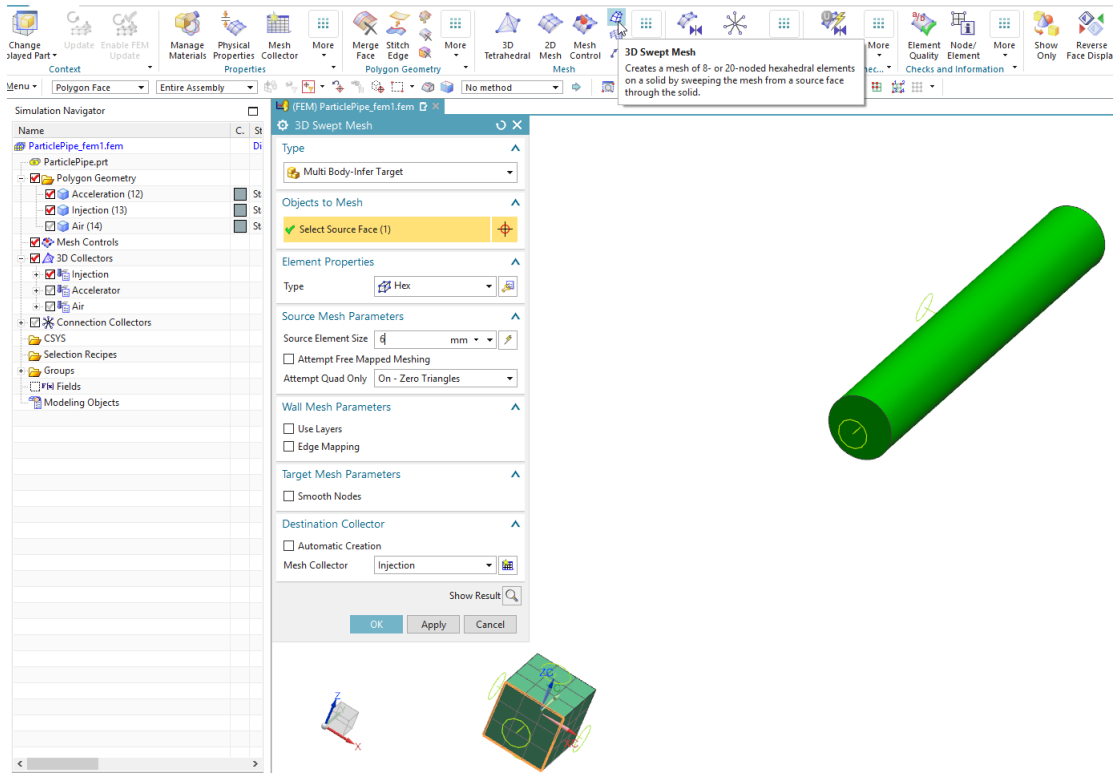
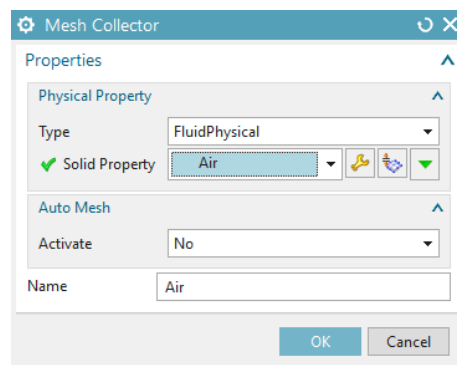
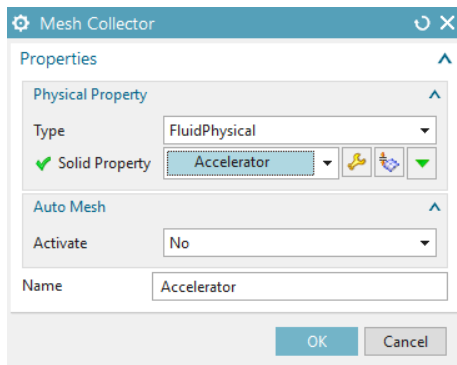


- Then, create a new Collector (right click on '3D Collectors'; and then 'New Collector'). Call This Collector *Injection*; and assign a Type 'Fluid Physical'. Create a new Physical called *Injection* and assign 'Air' as Material Repeat this process two more



times, i.e. create a new Collector called *Accelerator* and one called *Air* (also assign Type 'Fluid Physical' and Material 'Air')

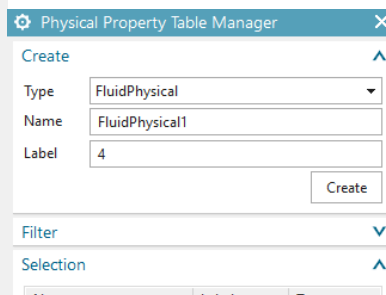
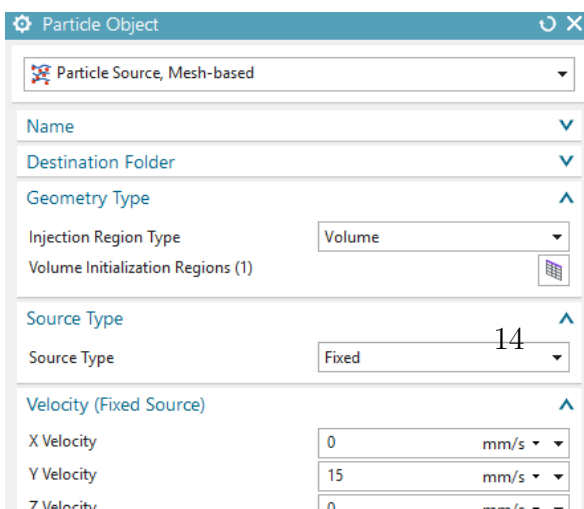
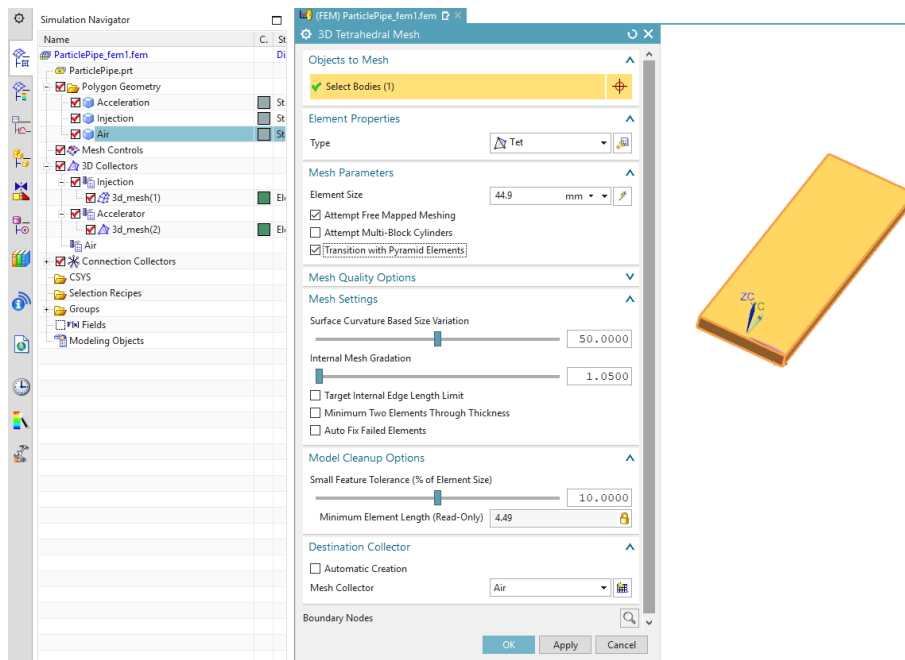
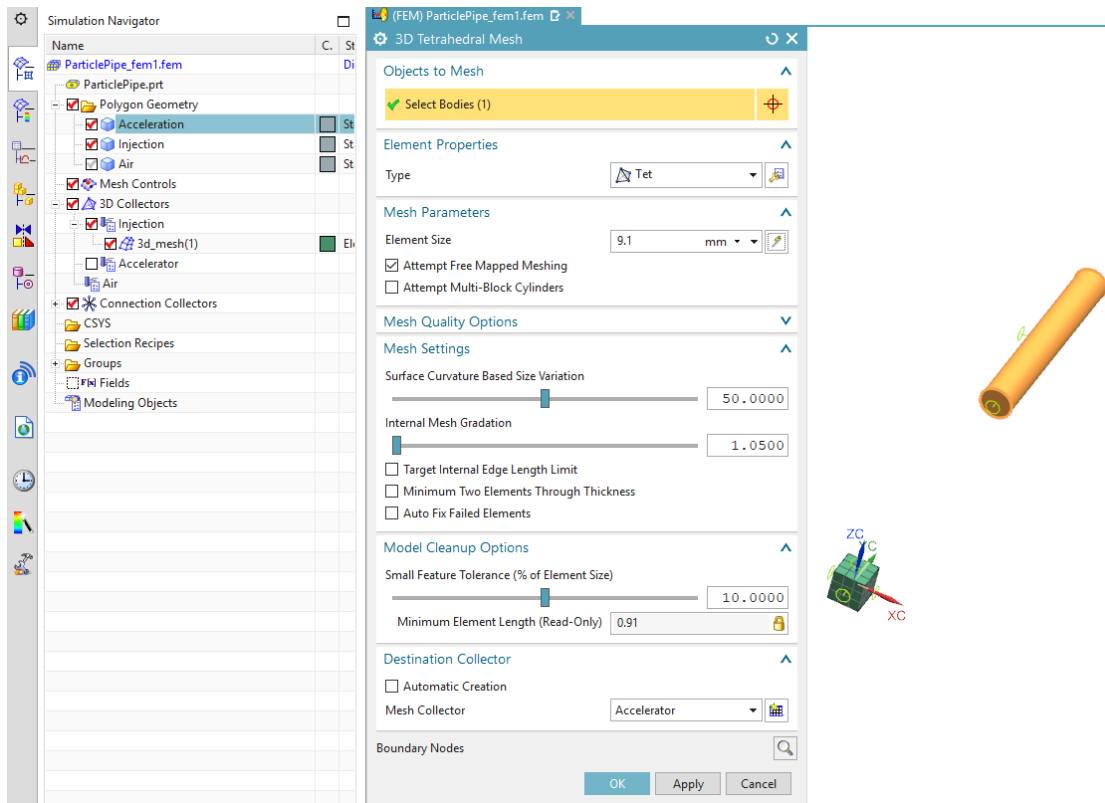
- Mesh the injection source with a '3D Swept Mesh' of size 6mm; herein set the option 'Attempt Quad Only' to 'On-Zero Triangles'. The meshing should result in a regular hex-mesh with a total of 27 Cells.
- Mesh the accelerator pipe with a '3D Tetrahedral' mesh of size 9.1mm; or a finer mesh.
- Mesh the surrounding air volume with a '3D Tetrahedral' mesh of size 44.9mm; or a finer mesh.

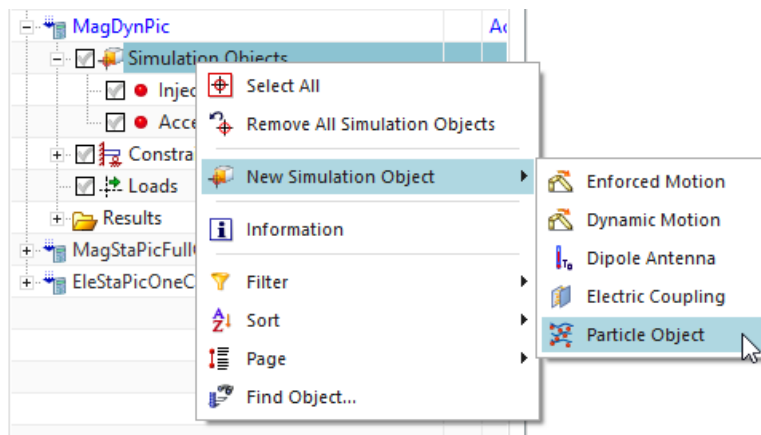
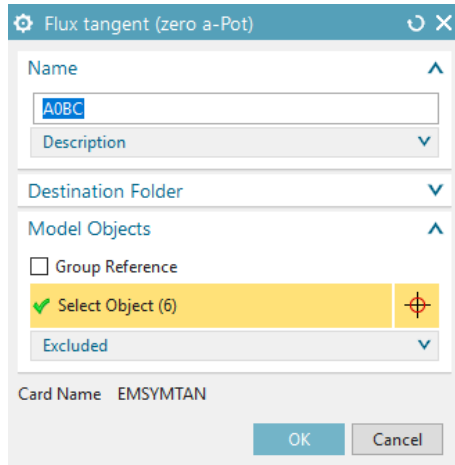


- In the top slider select 'Magnetics' and click 'Rename Meshes By Collectors'; then switch back to 'Home'.

8. Next, switch to the SIM environment (right click on the “.fem” file; then click 'Display Simulation')

- In the SIM environment, create a  $A = 0$  boundary condition. To do so right click on 'Constraints' and select 'Flux tangent'; then select the six faces of the air volume.
- Now, create an particle source. To do so right click on 'Simulation Objects', then 'New Simulation Object'; and select 'Particle Object'  
Then, specify the type to be 'Particle Source, Mesh-Based' and set the 'Injection Region Type' to 'Volume'.





Note.: The above source provides a total of 27 particles (1 Testparticle) with Mass 1 Kg and Charge  $1 \text{ A} \cdot \text{s}$  that travel with an initial velocity of  $15 \frac{\text{mm}}{\text{s}}$  in Y-direction.

Last, create the Volume Initialization Region. Thus, click on 'Create Volume Initialization Regions'. Then, select the Injection Physical and add it to the list. Press 'Ok'.

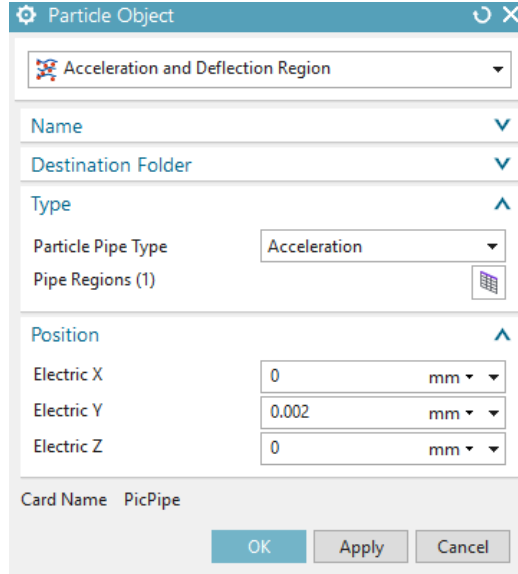
- Finally, create an accelerator pipe. Again right click on 'Simulation Objects' and 'New Simulation Object'; but now select 'Acceleration and Deflection Pipe'. Select the type to be 'Acceleration' and assign an Electric Y field of 0.002 (this provides an  $E_y$  field of  $0.002 \frac{\text{V}}{\text{m}}$ ).

Note.: The Acceleration and Deflection Pipes are provided to create constant electric and/or magnetic fields which can be used to accelerate and/or deflect particles. In the selected areas these fields are held constant (even despite other calculations that might be additionally created). Moreover, said fields are only visible for the particle solver. In a nutshell the pipes erase any field values in the specific regions and replace it with specified values for the Particle Solver.

- Press 'Solve' to solve the solution.

9. Switch to the Postprocessing environment (under "Results" double-click 'Magnetic')

- In the following the displacements and the velocities of the particles will be evaluated.



Thus, let us first specify the required theory and adjust it to this example. The (mechanical) motion of the particles is described by Newton's second Law

$$\mathbf{F} = m \mathbf{a} = m \ddot{\mathbf{x}}, \quad (12)$$

whereas the electromagnetic impact on the particles is given by the Lorentz Force / (assuming  $\mathbf{B} = 0$ , thus being the Coulomb Force)

$$\mathbf{F}_L = q \mathbf{E}. \quad (13)$$

Here,  $m$  is the Mass,  $q$  the charge and  $\mathbf{a}$  the acceleration. By simple integration we obtain

$$\mathbf{a}(t) = \frac{q}{m} \mathbf{E}, \quad (14)$$

$$\mathbf{v}(t) = \frac{q}{m} \mathbf{E}t + \mathbf{v}_0, \quad (15)$$

$$\mathbf{x}(t) = \frac{1}{2} \frac{q}{m} \mathbf{E}t^2 + \mathbf{v}_0t + \mathbf{x}_0. \quad (16)$$

As a consequence we deduce that an acceleration is only possible within the acceleration tube. In the field free Domains, Newton's first law applies on the particles: each particle moves with a constant velocity.

- Let us set appropriate plot options: for the displacements we choose spheres (click on 'Edit Post View'. Under 'Display' select 'Spheres'. Change the scale to 5.0 'mm'), and arrows for the velocities (click on 'Edit Post View'. Under 'Display' select 'Arrows')
- First, observe the overall particle motion. Thus, select 'Particle Velocities', click 'Animate'; and under 'Animate' select 'Iterations'. In total five timesteps (given by the Increments) are significant:

**Increment 1:** at time  $t = 0$  sec the particles are initialized in the source

**Increment 2:** at time  $t = 1$  sec the first particles exit the source

**Increment 12:** at time  $t = 11$  sec the first particles impinge the accelerator

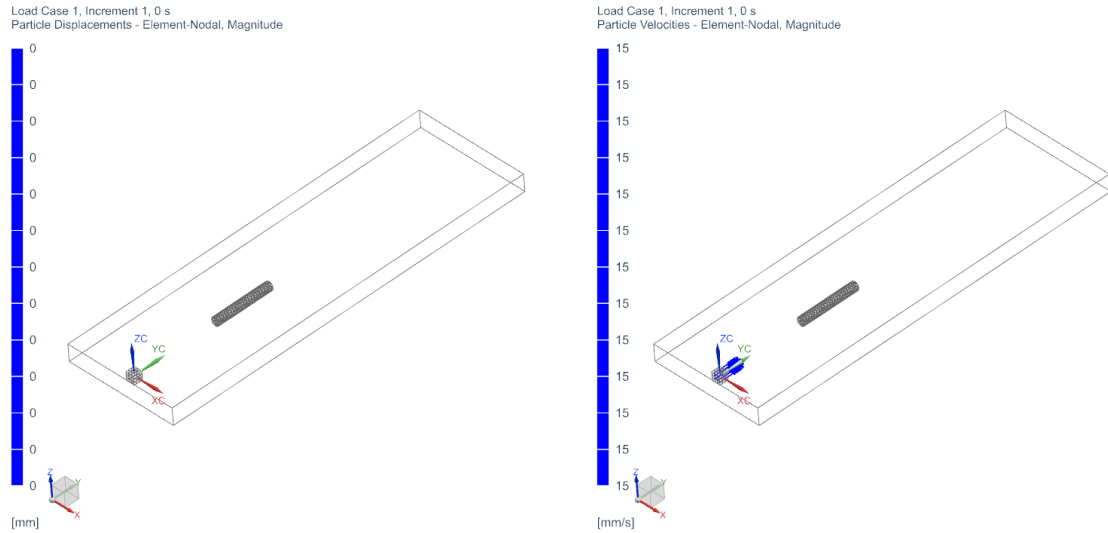
**Increment 16:** at time  $t = 15$  sec the first particles exit the accelerator



**Increment 31:** at time  $t = 30$  sec the last particles leave the air volume

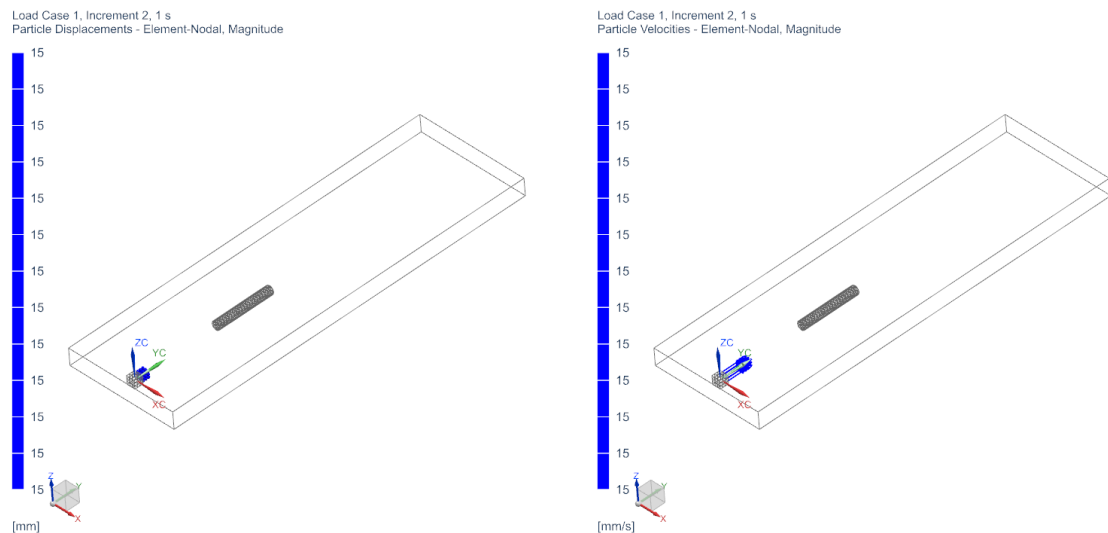
Let us now investigate these five incidents in detail.

- At  $t = 0$  sec the particles are initialized in the source. Observe that the displacement plot does not show anything. This is because there are no displacements present. The Velocities are set to  $15 \frac{\text{mm}}{\text{s}}$  as has been set.



It can be observed that there are 27 Particles present that are all initialized in the respective middle of each Cell within the Particle Injection (Remember that we have set 27 cells before). Consequently, the particles start in three fronts (with respect to the  $y$  direction). The first row starts at  $y = 5$ , the second at  $y = 0$  and the third at  $y = -5$ .

- At  $t = 1$  sec the particles in the first row have exited source. Now the displacement of the particles are visible. We observe displacements of 15 mm and Velocities of  $15 \frac{\text{mm}}{\text{s}}$ .

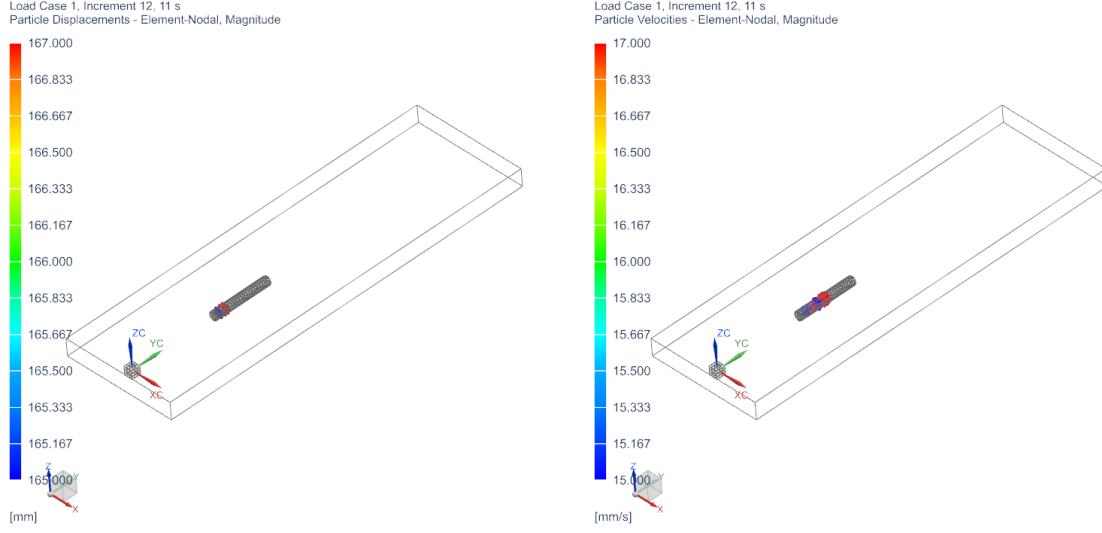


The observed Displacements and Velocities are expected. Since there is no electric

Field Present the velocity stays constant thus resulting in,  $x(t) = v_0 t = 15 \frac{\text{mm}}{\text{s}} \cdot 1\text{s} = 15\text{mm}$ .

Since we have chosen the distance between the particle source and the accelerator pipe to be 150 mm, we expect that the particles will travel 10 sec to the accelerator pipe. Thus the first row particles should have entered the accelerator pipe after 11 sec, i.e. at Increment 12.

- At  $t = 11$  sec the particles in the first row have entered the accelerator pipe. We observe that the (max) velocity has changed to  $17 \frac{\text{mm}}{\text{s}}$ . Moreover the (max) displacement is now 167 mm.



The observed Displacements and Velocities are again expected. Inside the accelerator pipe there is an electric field of  $2 \frac{\text{V}}{\text{mm}}$  present. Thus we obtain an acceleration of  $a(t) = 1\text{As} \cdot \frac{1}{\text{Kg}} \cdot 2 \frac{\text{V}}{\text{mm}} = 2 \frac{\text{mm}}{\text{s}^2}$ . With the starting velocity of  $15 \frac{\text{mm}}{\text{s}}$  this results in a velocity of  $17 \frac{\text{mm}}{\text{s}}$  after 1 sec in the acceleration pipe. Therefore, also the displacements are expected: the particles have traveled 150 mm to the pipe; here the velocity has changed to  $17 \frac{\text{mm}}{\text{s}}$ . Thus, after one timestep the particles are at 167 mm.

We have chosen the accelerator pipe to be 100 mm long. With the above acceleration we can calculate the time that the particles need to travel through this pipe:

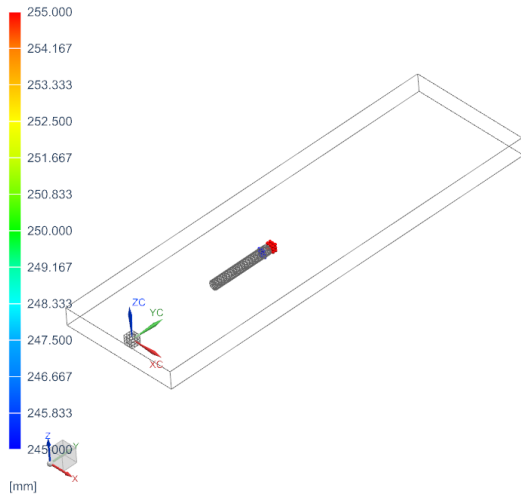
$$100\text{mm} = 1 \frac{\text{mm}}{\text{s}^2} \cdot t^2 + 15 \frac{\text{mm}}{\text{s}} \cdot t \quad \Rightarrow \quad t_+ = -\frac{15}{2}\text{s} + \sqrt{\frac{225+400}{4}\text{s}^2} = 5\text{s}.$$

During this time the velocity is expected to change by  $10 \frac{\text{mm}}{\text{s}}$ .

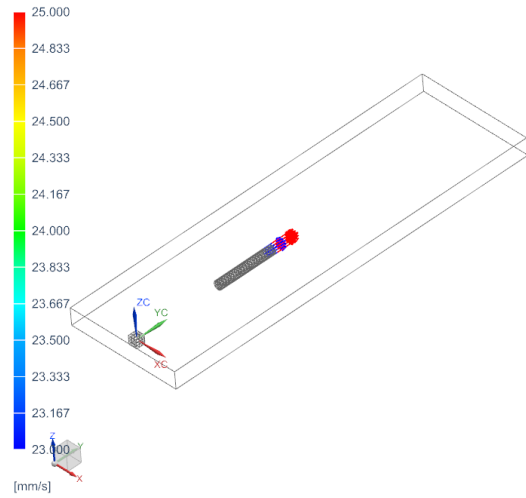
- At  $t = 15$  sec the particles in the first row left the accelerator pipe. We observe that the (max) velocity has changed to  $25 \frac{\text{mm}}{\text{s}}$ . Moreover the (max) displacement is now 225 mm.

The observed Displacements and Velocities are again expected. From the previous discussion we expect a velocity change of  $10 \frac{\text{mm}}{\text{s}}$ . With a starting velocity of  $15 \frac{\text{mm}}{\text{s}}$  this results in a velocity of  $25 \frac{\text{mm}}{\text{s}}$  after 15 sec. Therefore, also the displacements are expected: the particles have travelled 230 mm to the end of the pipe while being accelerated; and one second with  $25 \frac{\text{mm}}{\text{s}}$ . Thus, this results in a total of 255 mm.

Load Case 1, Increment 16, 15 s  
Particle Displacements - Element-Nodal, Magnitude



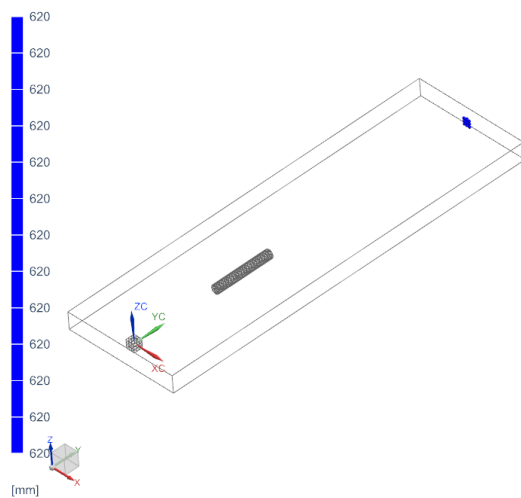
Load Case 1, Increment 16, 15 s  
Particle Velocities - Element-Nodal, Magnitude



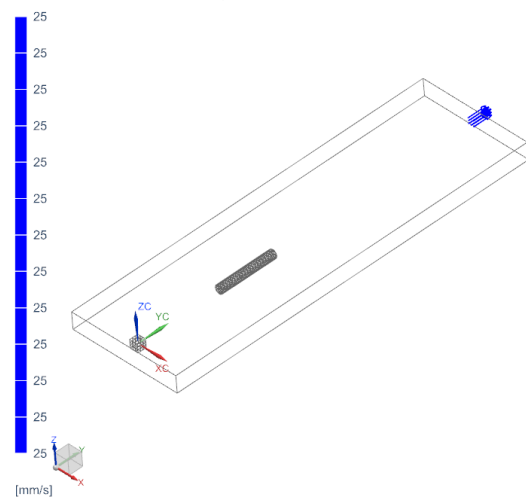
In the remaining 375 mm the particles should travel with a constant velocity of  $25 \frac{\text{mm}}{\text{s}}$  for 15 sec.

- At  $t = 31$  sec the last particles have left the air volume. We observe that the velocity is  $25 \frac{\text{mm}}{\text{s}}$ . Moreover the (max) displacement is now 620 mm.

Load Case 1, Increment 31, 30 s  
Particle Displacements - Element-Nodal, Magnitude



Load Case 1, Increment 31, 30 s  
Particle Velocities - Element-Nodal, Magnitude



- As a check we investigate the Magnetic Flux result; thus switch on the 'Magnetic Flux Density' result.

Remember that there is no external Magnetic Field Source (e.g. from a Coil or a Constraint etc.) in the simulation by default. The only source for a Magnetic field thus is by the particles. A moving particle  $i$  with velocity  $\mathbf{v}_i$  and charge  $q_i$  produces a Current

$$\mathbf{J}_i = q_i \mathbf{v}_i, \tag{17}$$

whereas a total current  $\mathbf{J}$  is just the sum of all single particle currents, and can enter as a source into (Magnetodynamic) Amperes Law

$$\nabla \times \mathbf{H} = \mathbf{J} = \underbrace{\mathbf{J}_s}_{=0} + \sum_i \mathbf{J}_i. \quad (18)$$

With this in mind, press 'Animate' (remember to switch on the 'Iterations' toggle); as a result we observe a (small) magnetic field in the wake of the particle bunch.


Note.: The selected 'Full Interaction Mode' allows the bi-directional coupling of sources and currents into Maxwells equations. In this example it thus enables to impose a current into Amperes Law.

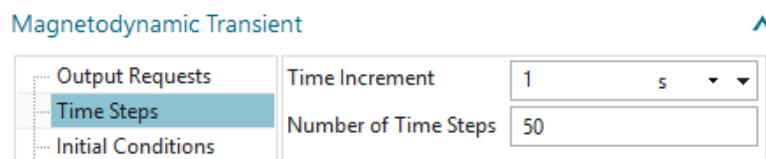
### 2.1.2 Single Particle – Magnetostatic

Abstract: A single particle starts with a velocity of  $15 \frac{\text{mm}}{\text{s}}$  from Coordinate  $\mathbf{x}_0 = (0, 0, 0)$  and heads towards an accelerator pipe with a constant electric field of  $1 \frac{\text{V}}{\text{mm}}$ , in order to be linearly accelerated. Here, a Magnetostatic solution with an uni-directional particle coupling is applied.

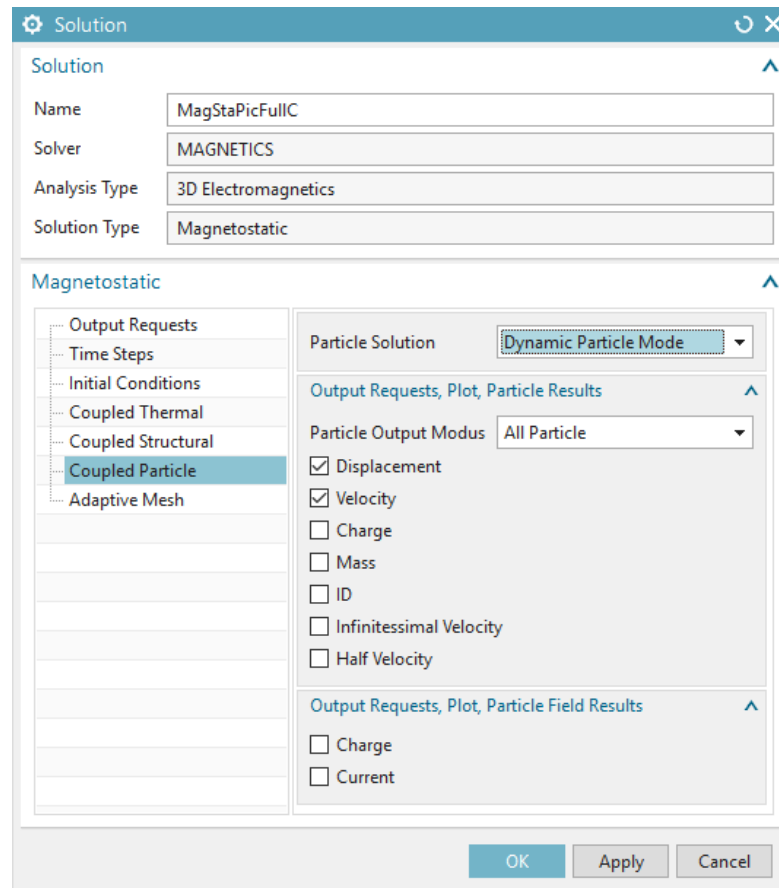
Estimated time: 0.1 h

Follow the steps:

1. In this example we directly start in the SIM environment. For the meshing we refer to steps 1-6 of the previous example.
2. In the solution bar press the 'Solution'  button; to create an additional solution.
  - As 'Solution Type' select 'Magnetostatic'
  - In register 'Output Requests' under 'Plot' again activate 'Magnetic Fluxdensity'
  - Under 'Time Steps' select a 'Time Increment' of 1 s with a total of 50 time steps



- In register 'Coupled Particle' set the 'Particle Solution' to 'Dynamic Particle Mode'. With this selection the Particle Solver is uni-directionally coupled with the field solver, meaning that the EM fields accelerate/deflect the particles; but the particles do not impose any sources and currents. Also set the 'Particle Output Mode' to 'All Particle' and select 'Displacements' and 'Velocity' as Output.
  - Press 'Ok'.
3. Switch to the new Magnetostatic solution.



- Within the new Magnetostatic solution, apply the previously used 'Flux Tangent' BC; as well as the accelerator pipe (i.e. by dragging said objects into the solution from the respective containers)
- Now, create a new 'Particle Source' (i.e. right click on 'Simulation Objects' and 'Particle Objects')
- Then, specify the type to be 'Particle Source' and set the 'Injection Region Type' to 'Coordinate'.

The above source provides one particle with Mass 1 Kg and Charge 1 A · s that starts at position  $\mathbf{x}_0 = (0, 0, 0)$  and travels with an initial velocity of  $15 \frac{\text{mm}}{\text{s}}$  in Y-direction.

Note: In contrast to the previous mesh-based source, this source is coordinate based. Thus, the previous creation of a CAD geometry and subsequent meshing is not required for this type of source.

4. Press 'Solve'.

5. Switch to the 'Results' environment.

- The displacements and velocities of the particle behave similar as in the previous example. However, in this simplified scenario they are of course exact, meaning that there is no spread (we omit the calculation and refer to the previous example).
- As a last check we investigate the Magnetic Flux result again; thus switch on the 'Magnetic Flux Density' result. We observe that the Magnetic Flux Density is zero

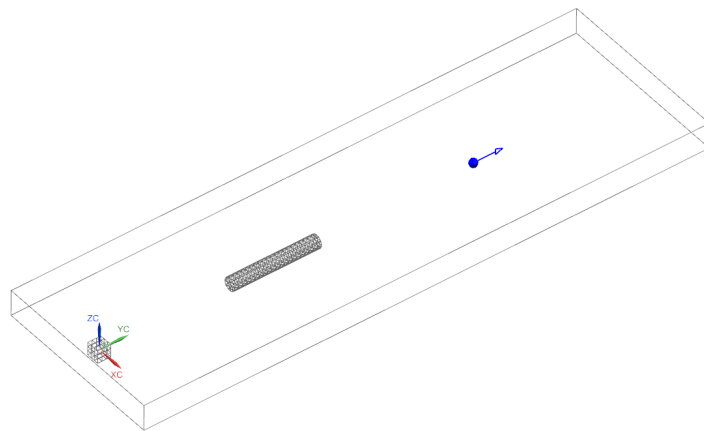
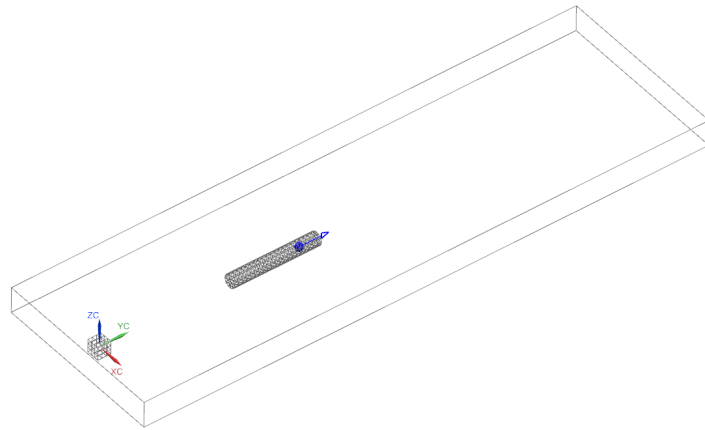
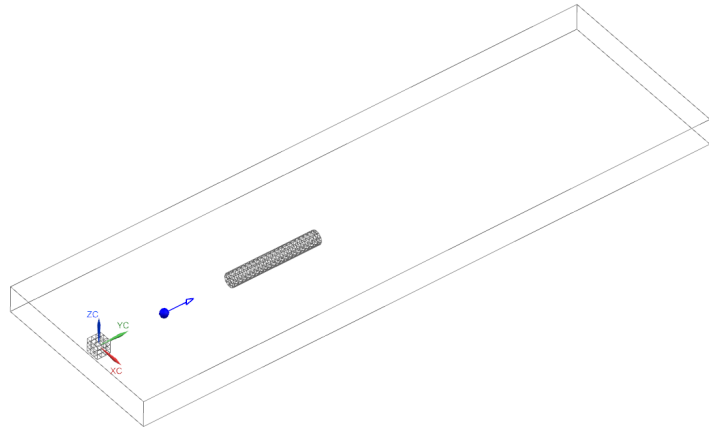
everywhere. This result is expected, since we have switched on the uni-directional coupling, meaning that the particles do not impose any sources or currents on the field solver, meaning

$$\nabla \times \mathbf{H} = \mathbf{J} = \underbrace{\mathbf{J}_s}_{=0} = 0. \quad (19)$$


### 2.1.3 Thermal Particle Initialization – Electrostatic

Abstract: Two particle sources are initialized with 1 Million particles, respectively. The particles are initialized with thermal velocities, i.e. each velocities absolute value corresponds to a particle in an ideal gas at 20.000°C; with directions choosen randomly. The sources are initiated at Coordinates  $\mathbf{x}_0^1 = (0, 0, 0)$  and  $\mathbf{x}_0^2 = (0, 0, 0)$  and an accelerator pipe with a constant electric field of  $1 \frac{\text{V}}{\text{mm}}$  is employed. Here, an Electrostatic solution with an uni-directional particle coupling is applied.

Estimated time: 0.25 - 0.5 h (hardware dependent)

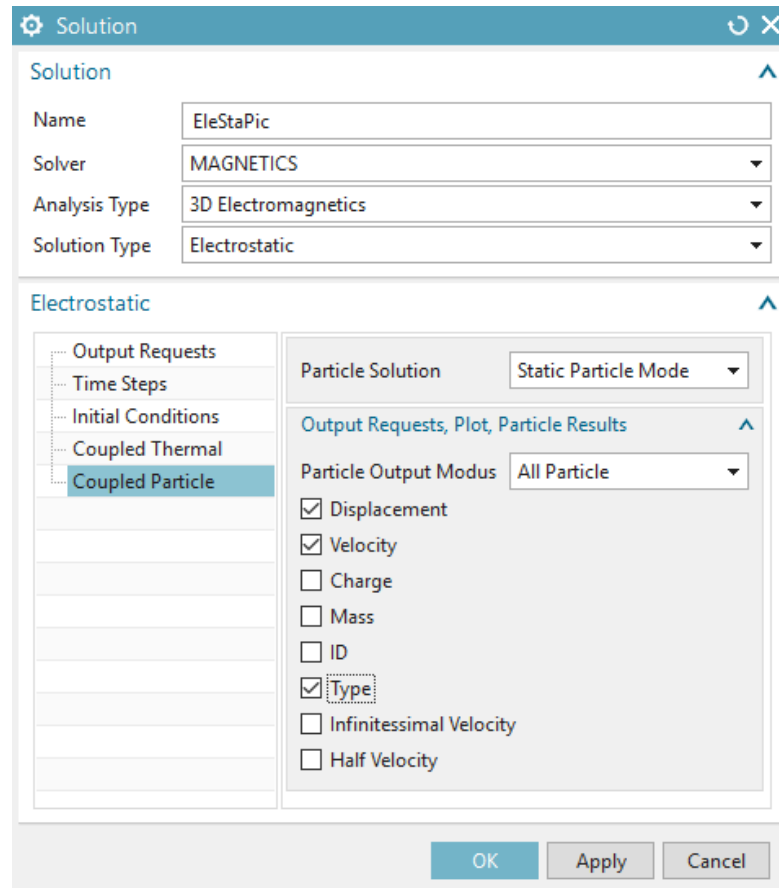


Follow the steps:

1. In this example we directly start in the SIM environment, again. For the meshing we refer to steps 1-6 of example 1.
2. In the solution bar press the 'Solution'  button; to create an additional solution.
  - As 'Solution Type' select 'Electrostatic'
  - In register 'Output Requests' under 'Plot' we deselect all Output.
  - Under 'Time Steps' select a 'Time Increment' of 1 s with a total of 100 time steps



- In register 'Coupled Particle' set the 'Particle Solution' to 'Static Particle Mode'.



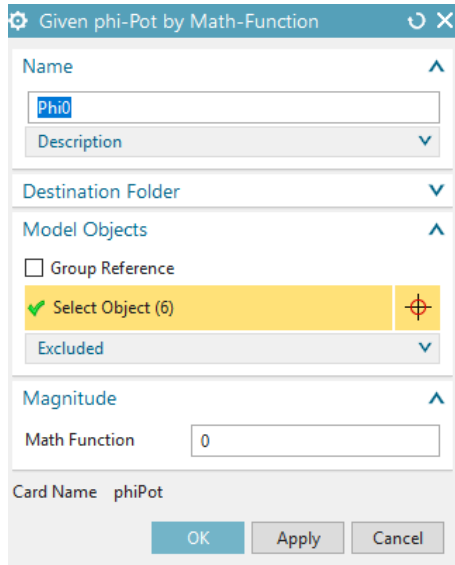
With this selection the Particle Solver is uni-directionally coupled with the field solver, meaning that the EM fields accelerate/deflect the particles; but the particles do not impose any sources and currents. In addition the Fields are only passed to the particle solver at initial time, i.e. one time only. Also set the 'Particle Output Mode' to 'All Particle' and select 'Displacements', 'Velocity' and 'Type' as Output.

- Press 'Ok'.

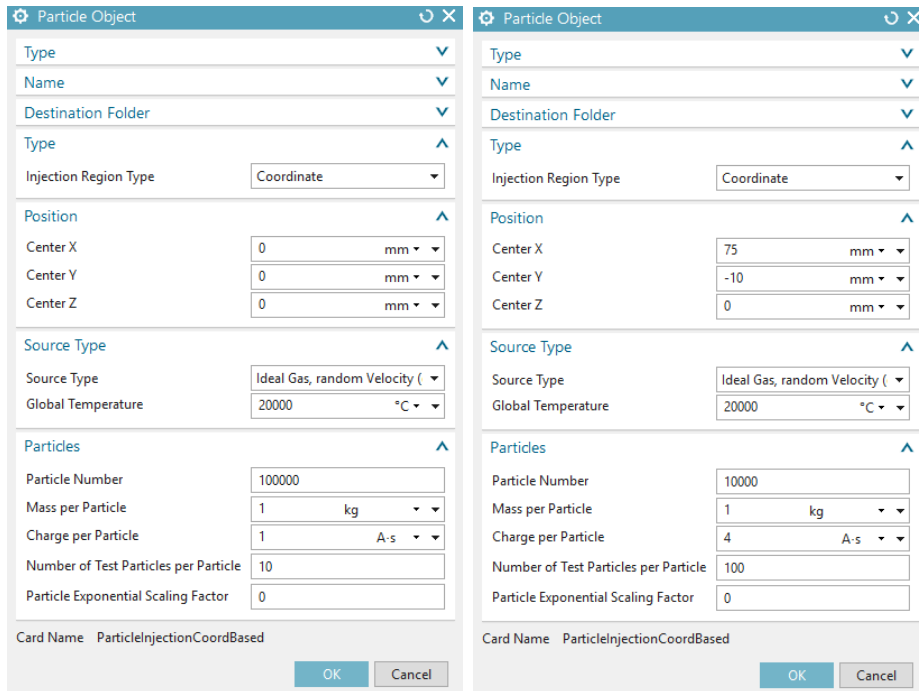
### 3. Switch to the new Electrostatic solution.

- In the SIM environment, apply the acceleration pipe from the previous examples (i.e. drag it into the 'Simulation Objects').
- Now, create a  $\varphi = 0$  boundary condition. To do so right click on 'Constraints' and select 'Given phi-Pot by Math-Function'; then select the six faces of the air volume.
- Now, create two new 'Particle Sources' (i.e. right click on 'Simulation Objects' and 'Particle Objects')





- Then, specify the types to be 'Particle Source' and set the 'Injection Region Type' to 'Coordinate', respectively (apply  $\mathbf{x}_0^1 = (0, 0, 0)$  and  $\mathbf{x}_0^2 = (0, 75, -10)$ ). Under 'Velocity Distribution' select 'Full Thermal' with a 'System Temperature' of 20.000°C.



The above sources provide one Million particles each, that are initialized with thermal distributions. In the first source 100.000 Particles with 10 Testparticles each are initialized with Mass 1 Kg and Charge 1 A · s, whereas in the second source 10.000 Particle with 100 Testparticles each are initialized with Mass 1 Kg and Charge 4 A · s.

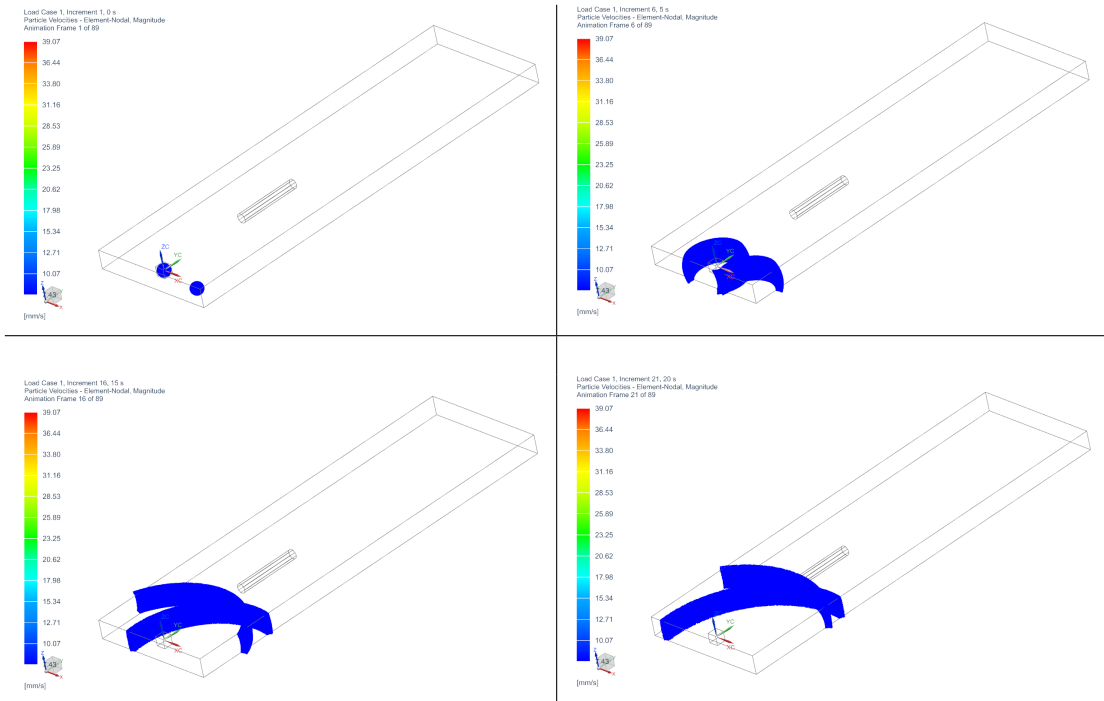
Note: Testparticles are used for statistic averaging (i.e. to obtain a certain Liouville average within a Plasma Simulation). Thus, the effect of Testparticles is similar

to that of smearing.

4. Press 'Solve' (the Solve will take some time).

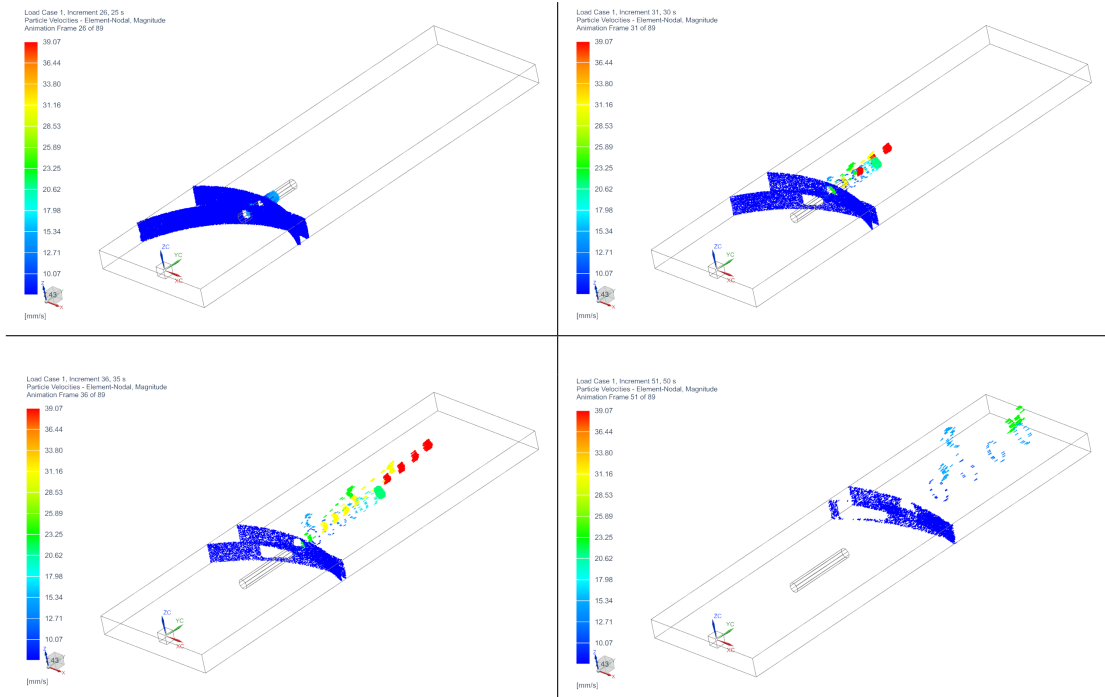
5. Switch to the 'Results' environment.

- In the following we will solely investigate the particle velocities, thus click on the 'Particle Velocities' result. To obtain a decent plot select 'Edit Post View' and switch the 'Color Display' to 'Arrows'. Then switch the 'Size' to 'Result' and select a suiting 'mm' value, such as 10 mm.
- Then start an Animation, i.e. press the 'Animate' button ( and remember to switch on the 'Iterations' toggle). The plotting process may take a while since many particles to be plot are present. When the Animation is finished, observe the particle evolution.
- Due to the fact that very many particles are present, we observe a wave-like behaviour of non-interacting particles. In particular we see two “furball”-like structures at time  $t = 0$  sec around the two sources, from which wave-fronts are spherically expanding with ongoing time.



Until 20 sec the particles from the two sources move at the same absolute velocities (but with different directions). This is expected, since the velocities are just the starting velocities; and are not changed, due to the absence of any forces. Indeed this behaviour is the same as in the previous examples (just with more particles).

- After (roughly) 20 sec the first particles start impinging on the accelerator pipe, and are accelerated by the electric field (of  $2 \frac{V}{mm}$  in Y-direction). After 30 sec we obtain a difference in the change on the velocities of the particles from the two sources. The particles from the second source at  $\mathbf{x}_0^2$  are accelerated four times faster than the particles from the first  $\mathbf{x}_0^1$ . Moreover, said particles are moving slightly to the left.



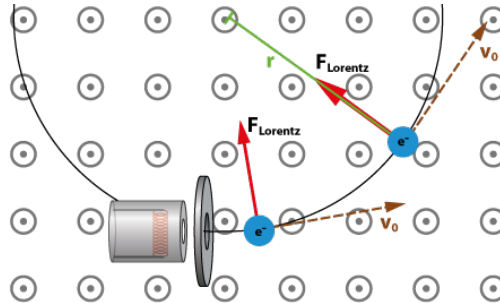
Again the observed results are expected. Recall that the particles from the second source were initialized with four times larger charges  $4 \text{ A} \cdot \text{s}$ . Due to  $\mathbf{a}(t) = \frac{q}{m} \mathbf{E}$  they are thus accelerated four times faster. In addition the second particle source has been offset by 75 mm right to the first particle source. As a result the particles are impinging at the accelerator pipe under a larger angle and also exit the accelerator pipe under a larger angle.

The tutorial is complete

## 3 Tutorial 2

### 3.1 Electrons in Homogeneous Magnetic Field

Goal of this simulation is finding the radius of an electron in an homogeneous magnetic field and checking the result against analytic.



#### 3.1.1 Analytic Solution

The Lorentz force is always perpendicular to the direction of motion of the electrons when electrons move in the B field. Further, it is the only acting force. Thus, the Lorentz force is the centripetal force necessary for a circular path. So it applies

$$\mathbf{F}_{Lorentz} = \mathbf{F}_{Zentripetal}$$

Inserting of  $\mathbf{F}_{Lorentz} = e \cdot v_0 \cdot \mathbf{B}$  and  $\mathbf{F}_{Zentripetal} = m_e \frac{v_0^2}{r}$  leads to

$$e \cdot v_0 \cdot \mathbf{B} = m_e \frac{v_0^2}{r}$$

Solving for r yields the formula for the radius of the circular path:

$$r = \frac{m_e \cdot v_0^2}{e \cdot \mathbf{B}}$$

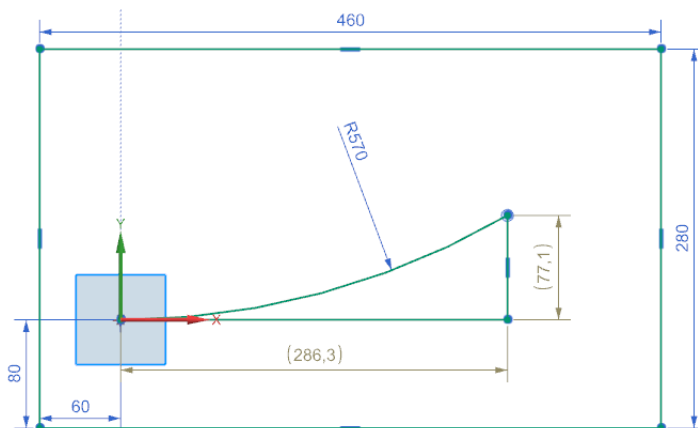
This radius is also called 'Larmor Radius'.

Reference:

[www.didaktik.physik.uni-muenchen.de/elektronenbahnen/b-feld/B-Feld/Auswertung.php](http://www.didaktik.physik.uni-muenchen.de/elektronenbahnen/b-feld/B-Feld/Auswertung.php)

#### 3.1.2 Simulation Model

The picture below shows a sketch of the simulation model. The model we use to simulate for the electron deviation uses following properties.



- $B = 0.001 \text{ T}$
- $m_e = 9.109 \times 10^{-31} \text{ Kg}$
- $V_0 = 1 \times 10^8 \text{ m/s}$
- $e = 1.602 \times 10^{-19} \text{ C}$
- Analytic radius result:  $r = 570 \text{ mm}$
- Time Step:  $1 \times 10^{-10} \text{ s}$
- Number of Time Steps: 30
- Expected total displacement: 300 mm
- Expected x,y displacement (from sketch): 286.3 mm, 77.1 mm

All particles start from the coordinate center in x direction. A circle with radius 570 mm and arc length 300 mm shows the way particles should go as expected from the analytic result. With an element size of 5 mm the electrons will jump over two element with each time step.

### 3.1.3 Simulation Results



The simulation result is as follows.

- Total displacement: 297.2 mm (deviation 0.9%)
- x displacement: 286.8 mm (deviation 0.17%)
- y displacement: 77.74 mm (deviation 0.82%)

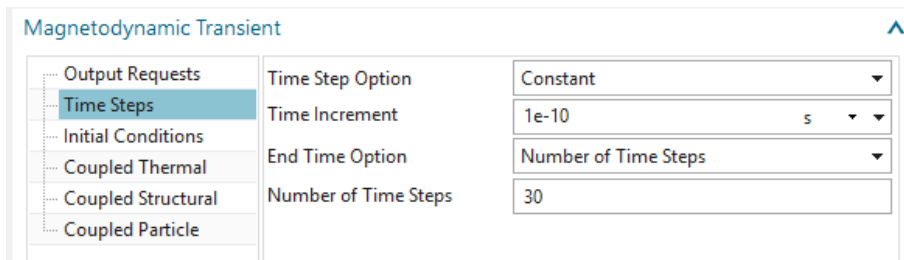
Small deviations occur between simulation and analytics. These can come from the bidirection effect that is not included in the analytic formula. This may also come from the quite large jump at each time step.

### 3.1.4 Tutorial

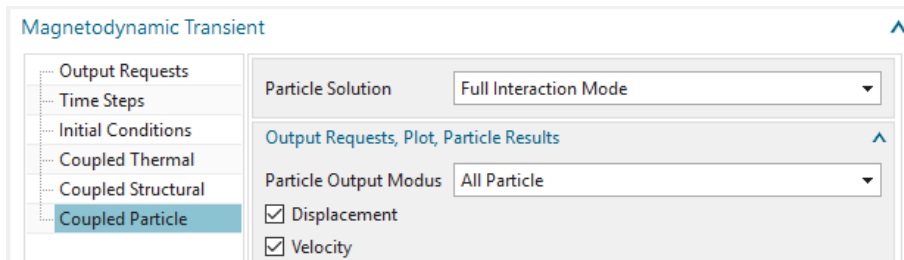
Necessary time: 20 min. The model is already build and ready to solve. To perform this simulation follow these steps:

1. Download the model files for this tutorial from the following link:  
<https://www.magnetics.de/downloads/Tutorials/11.CouplParticle/11.2LarmorRadius.zip>
2. Start the Program Simcenter  (or NX). Use Version 12 or higher.
3. In Simcenter, open  the file 'LarmorRadiusCheck\_sim1.sim'.

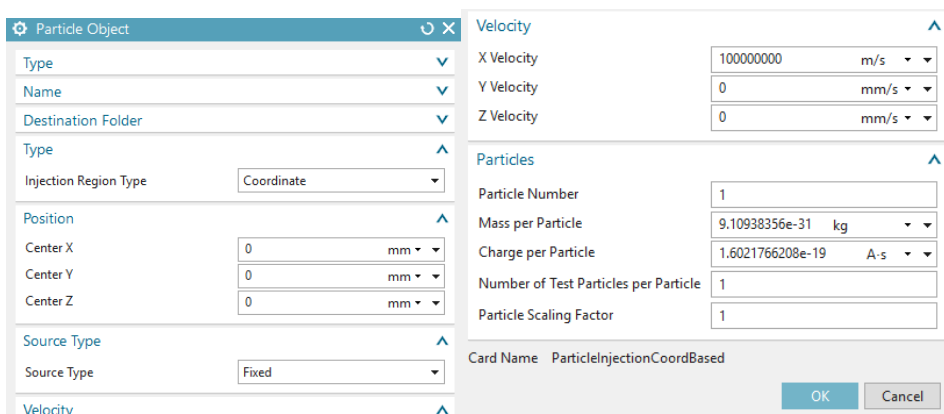
4. Edit the solution 'MagDynPicBi' and check register 'Time Steps'.



5. check register 'Coupled Particle'.



6. Check the simulation object 'Particle Injection(1)'

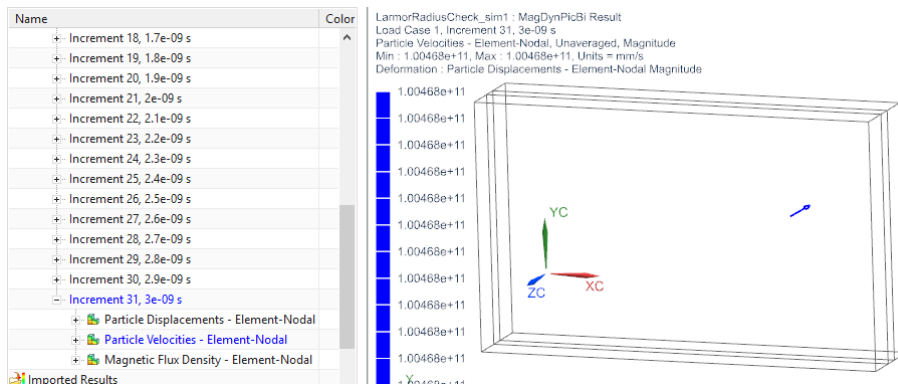


7. Solve the solution: Click on  and OK. The solver will run about 3 min.

8. after the solve has finished open the 'Particle Displacement' result.

9. set the time step to the last and read the result for x and y displacement. These values should be as discussed above.

10. check for other results as desired. Following picture shows the velocity at the last step.

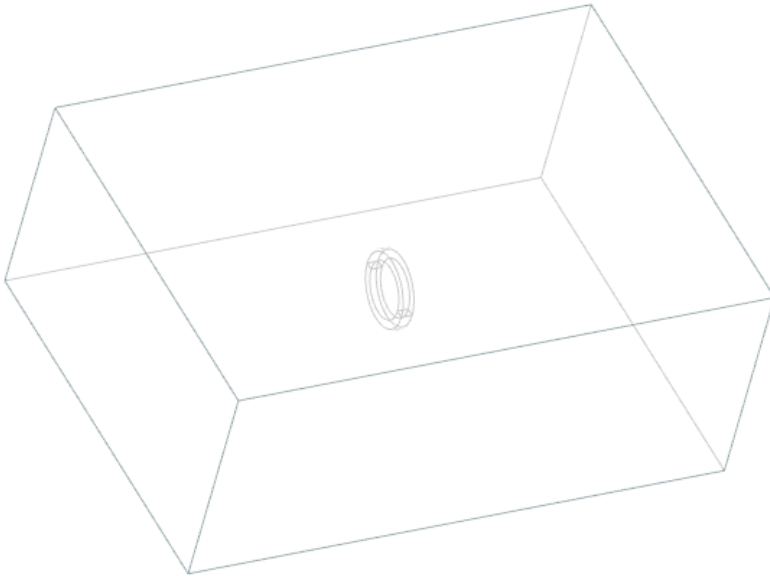


This tutorial is complete.

## 4 Tutorial 3

### 4.1 Filament Thermionic Emission



This chapter shows different models how thermionic emission can be simulated with NX Magnetics. They all are shown with a filament model.



The following thermionic emission models are available:

- Richardson-Dusham's Law
- Child's Law

The models are already build and ready to solve. Their content and the used features allow getting insight in the methods for thermionic emission usage.

1. Download the model files for this tutorial from the following link:  
<https://www.magnetics.de/downloads/Tutorials/11.CouplParticle/11.3Filament.zip>
2. Start the Program Simcenter  (or NX). Use Version 13 or higher.
3. Open  the file 'Filament\_sim1.sim' and click OK.

#### 4.1.1 Richardson-Dusham's Law

Check the solution 'ElSta\_Pic\_Richardson'. It shows the usage of thermionic emission using Richardson-Dusham's model. Check the simulation object 'Particle Object Richardson'.



⚙ Particle Object
↺ ✕

Type ▼

Name ▼

Destination Folder ▼

**Geometry Type** ▲

Injection Region Type Surface ▼  
 Surface Initialization Regions (1) 📅

**Source Type** ▲

Source Type Richardson-Dusham ▼

**Velocity (Richardson)** ▲

Type Thermionic Source, direction ▼  
 Material Quantum Correction Factor 0.5 ▼  
 Material Escape Velocities [ElectronVolt] 1 ▼

**Particles** ▲

Particle Number 1  
 Mass per Particle 9.10938356e-31 kg ▼  
 Charge per Particle 1.6021766208e-19 A-s ▼  
 Number of Test Particles per Particle 1  
 Particle Exponential Scaling Factor 0  
 Spatial Distribution Random Positions ▼

Card Name ParticleInjectionMeshBased

OK
Cancel

### 4.1.2 Child's Law

Check the solution 'ElSta\_Pic\_Child'. It shows the usage of thermionic emission using Child's model. Check the simulation object 'Particle Object Child'.

Particle Object

Type

Name

Destination Folder

Geometry Type

Injection Region Type: Surface

Surface Initialization Regions (1)

Source Type

Source Type: Childs Law

Velocity (Child)

Type: Thermionic (cold Cathode R)

Voltage Difference in Cathode Ray: 30000 V

Distance between Anode and Cathode: 0.001 mm

Particles

Particle Number: 1

Mass per Particle:  $9.10938356 \times 10^{-31}$  kg

Charge per Particle:  $1.6021766208 \times 10^{-19}$  A·s

Number of Test Particles per Particle: 1

Particle Exponential Scaling Factor: 0

Spatial Distribution: Fixed Positions

Card Name: ParticleInjectionMeshBased

OK Cancel

This tutorial is complete.

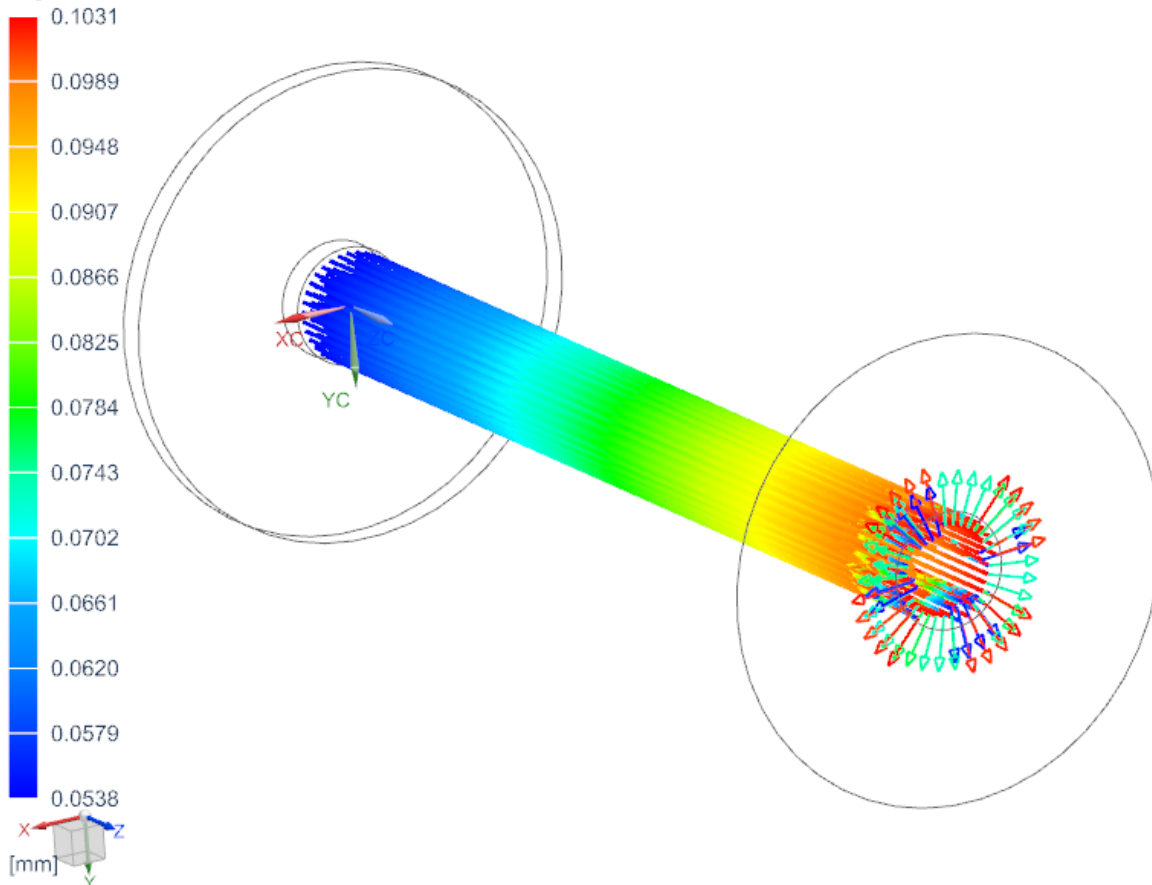
## 5 Tutorial 4

### 5.1 Diverging Electron Beam

When modeling the motion charged particles (electron beam) at a high electric current, the electric field due to the space charge of the charged particles significantly affects their trajectories. Goal of this analysis is to analyse the radius change of a electron beam.

Download the model files for this tutorial from the following link:

<https://www.magnetics.de/downloads/Tutorials/11.CouplParticle/11.4DivergingElectronBeam.zip>



To run the model change the number of particles from 62 to 62400.